Total Acquisition on Graphs

![Graphs Diagram]

The weight is now at one vertex.
Definitions and Rules

• A graph $G$ is defined as a set of vertices and edges.
• Before any movement has occurred on $G$, each vertex (lattice point) has a weight of 1.
• To move weight from one vertex to another, two rules will be followed.
• Rule 1: To move weight from $u$ to $v$, the weight at $v$ must be greater than or equal to the weight at $u$.
• Rule 2: When moving weight from $u$ to $v$, all weight from $u$ must be moved together.
• Such a move is called a total acquisition move.
• $a_t(G)$ is the total acquisition number. The total acquisition number of a graph $G$, is the minimum number of vertices with positive weight after a sequence of total acquisition moves.
• We study $a_t(G)$ when $G$ is an $mxn$ grid.
Let’s play
How many tiles are needed in any rectangle?

The maximum weight for any one tile is 16 since the maximum degree is 4 and $2^4 = 16$.

To find the minimum number of tiles needed, multiply the two dimensions together and divide by 16 and round up.

Therefore:

$$a_t(P_m \square P_n) \geq \left\lfloor \frac{mn}{2^d} \right\rfloor$$

when $m$ and $n$ are natural numbers and $d$=highest degree of any vertex.

Here’s where things get interesting…
Visual Index for Optimal Tilings on Rectangles
What about big rectangles?

Theorem: for $m>6$ and $n>6$ the total acquisition number for the $mxn$ grid is $\left\lfloor \frac{mn}{16} \right\rfloor$. 
What about rectangles that have one small dimension and one large dimension?

For $m=1,2,3,4,6$ it’s not so bad.

For $m=5$ things get really hard…….
Tilings for 5 \times n that contain 6 special vertices
All possible Double Tiles that take a total of 12 special vertices

The next tiling going to the right will contain at most 5 special vertices
Digraph of how tiles fit together in a 5xn grid
5xn grids have a different lower bound for the total acquisition number.

\[ a_t \geq \left\lfloor \frac{11n}{32} \right\rfloor \]