Shall We Play a Game?

The Match Game

- This is a two player game. Imagine a pile of 10 matches.
- Players take turns removing either 1, 2, or 3 matches from the pile.
- The player who removes the last match wins!

Solution to The Match Game

Let's think about a pile of matchsticks.
- Pile size 1, 2, or 3: All matches can be taken and you win.
- Pile size 4: You leave 1, 2, or 3 matches for the other person and they win.
- Pile size 5, 6, or 7: You can bring the pile down to 4, which forces the other person to lose.
- Pile size 8: You leave 5, 6, or 7 which allows the other person to leave you with pile size 4.

What will your strategy be if you have 14 matchsticks? 17? 20?

Win/Loss Positions and The Match Game

Rules for finding Win/Loss positions:
1. All end positions are L-positions.
2. From every W-position there is at least 1 move to an L-position.
3. From every L-position, you can only move to a W-position.

- 0 matches is an L-position because it is an end position.
- 1, 2, and 3 matches are all W-positions because they can be taken to 0 matches (an L-position).
- 4 matches is an L-position because you can only get to 1, 2, or 3 matches (which are all W-positions).
- This pattern continues, giving us the result that multiples of 4 are L-positions and anything else is a W-position.

Winning Strategy vs. Good Strategy

If you were able to solve The Match Game, you know that there is a way for one player to always win. This is known as a winning strategy.

Can you find a winning strategy for many of the games you know, like Tic-Tac-Toe, Rock-Paper-Scissors, or Sorry?

Why or why not?

Not all games have a Winning Strategy...

Impartial Combinatorial Games

Definition
- 2-person, alternate play
- Perfect information
- Not a game of skill
- No chance moves
- No dice or cards
- Moves same for both players (symmetric)
- Not chess or checkers
- Only Win or Lose outcome (no tie)
- Last person to move wins

Examples of Impartial Combinatorial Games
- The Match Game
- Nim (or next slide)
- The Game of Queues
- Othello
- The Game of Go

The Secret of NIM

- Now imagine three piles of matches. On your turn you may remove any number of matches from any single pile. Again, the player who takes the last match wins.
- Play this game with a friend or classmate using different pile sizes.
- Now find a winning strategy!

- Amazingly enough, you can always find a winning strategy for every impartial combinatorial game.
- If you can find the winning strategy for NIM, you have the tools necessary to find winning strategies for any impartial combinatorial game.